Lesson plan

Multiplicative reasoning

Contents

1. Rationale

When solving proportional problems, students often have just one strategy, most commonly an additive approach. This lesson uses the context of hotel stays and a double number line to explore different ways of solving these problems using both additive and multiplicative approaches. Students are asked to think about the approaches in terms of efficiency for solving proportion problems.

Using representations to provide access to the mathematical structure of a problem is a key principle of teaching for mastery (Key Principle 1). In this lesson, the double number line is used to illustrate and compare the thinking that relies on addition with that which relies on multiplication.

Proportion problems can crop up in many places in the curriculum, e.g. exchange rates, distance–time, rates of pay, conversion between units and similar triangles. This lesson could be used effectively in any of these different places in the curriculum, emphasising that, while the context may change, the approach stays the same. This helps to support students in developing a **connected understanding** of mathematical topics, which is an important aspect of teaching for mastery.

2. GCSE curriculum

Ratio, proportion and rates of change

R10 solve problems involving direct and inverse proportion

R11 use compound units such as … unit pricing

3. Lesson objectives

- Understand the multiplicative relationship between two quantities (non-calculator)
- Solve multi-step currency or unit conversions problems (calculator)
- Understand how to use representations to provide insight into solving problems

4. Starting points

This lesson assumes that students have some experience of solving proportional problems. It is likely that they will use an additive approach rather than a multiplicative approach.

5. Research questions

Pedagogic focus

How is the lesson developed and brought to a close in ways that values and builds upon what students already know?

Maths focus

What evidence do you observe of students' prior learning and how do they work with or modify this?

6. Lesson structure

Activity	Time (min)	Description/Prompt	Materials
Introduction	10	Introduce the context of hotel stays. Ask students to solve a proportion problem, first working alone and then in pairs.	Mini whiteboards Slide 2
Discuss 1	15	Discuss the different approaches that could be used to find the cost of an 11-night stay, illustrating these on a double number line. Start by discussing the methods that were used by the students, and move on to discuss the alternative approaches given on Slide 4.	Slides 3–11
Explore 1	15	Ask students to work in pairs to determine the cost of different lengths of stay in two currencies (Euros and South African Rand).	‘Cost of stay’ handout Calculators Slide 12
Discuss 2	10	Once students have completed the task, discuss the different student approaches.	Slide 13
Explore 2	10	Introduce a new currency (East Caribbean Dollars). Explore its relationship to Euros before making links back to the context of cost of hotel stays.	Mini whiteboards Slides 14–16
Explore 3	10	Ask students to work on a proportion problem using the new context of a train journey.	Mini whiteboards Slide 17
Review	10	Summarise the different relationships explored during the lesson, emphasising the multiplier in proportional relationships.	Slide 18
Practice question	10	Ask students to answer an exam question and after a few minutes discuss their thinking.	Slide 19

7. Teacher guidance

Introduction

Aim	Students solve a proportion problem in the context of charges for hotel stays
Materials	Mini whiteboards
Slides	Slide 2
Time	10 minutes

In this section of the lesson, students are asked to work on a proportion problem, first on their own, and then in pairs. This provides an opportunity to recognise students' different starting points. **Building on their existing knowledge** is key to a mastery approach (Key Principle 2).

What the students might do and what you might do

Introduce the context of the owner of a hotel charging the same price per night, regardless of how long someone wants to stay (this is an important assumption in this task) and tell students that the charge for a 2-night stay is £50.

Ask half the students to find the cost of a 5-night stay, and ask the other half of the class to find the cost of an 11-night stay. Give students a few minutes on their own to think about how to tackle the problem, before pairing students that have been working on a different number of nights.

Ask pairs of students to explain their approach to each other and agree on an approach for finding the two lengths of stay. They should write their method for finding a solution on their mini whiteboards.

As students work through the problem, identify any students who use one of the four approaches described on Slide 4. These students can be called on when these methods are covered in Discuss 1.

Discuss 1

Aim	To explore different ways of mathematical thinking
Materials	Whiteboard
Slides	Slides 3–11
Time	15 minutes

This section of the lesson discusses the proportion problem that students worked on in the previous section. It focuses on the relationship between the cost of 2 nights and 11 nights rather than the relationship between 2 nights and 5 nights, as the latter lends itself more easily to an additive approach involving doubling and halving.

A double number line is used to illustrate students' own approaches and to examine alternative approaches that may not have been used within the class. Mathematical representations, such as double number lines, allow students to develop an understanding of **multiplicative structure** (Key Principle 1).

Encourage students to think about the different approaches in terms of efficiency for solving proportion problems. Students may choose to change the way they approach proportion problems if they see a way of thinking that is more efficient or easier to understand. It is important to **value and make sense of students' different ways of thinking** (Key Principle 2).

What the students might do and what you might do

Slide 3 Bring the class together and ask students to explain their method for finding the cost of an 11-night stay. Capture the different methods on the double number line provided on the slide.

It is common for students to use an additive approach. For example, students might multiply the cost of a 2-night stay by 5 (£50 × 5 = £250), find the cost of 1 night (£50 ÷ 2 = £25) and then add these two values together (£250 + £25 = £275).

Slides 4–8 Slide 4 shows four different approaches to finding the cost of an 11-night stay. Slides 5–8 illustrate each of these approaches using an animated double number line. These approaches can be discussed in any order.

Slide 4 Ask the class about the four approaches, relating them to approaches you have already discussed when capturing approaches from the class on the double number line in Slide 3.

Ask students who have used a similar method to describe the thinking behind the method, using animated slides 5–8.

Slide 5 uses an additive approach. Ava has added the cost of 2 nights twice to find the cost of 4 nights (£100). She has then halved the cost of 2 nights to find the cost of 1 night (£25). Adding these together gives the cost of 5 nights (£125).
To find the cost for 11 nights, she has used 5 nights + 5 nights + 1 night (£125 + £125 + £25 = £275).

Slide 6 uses the unitary approach. Bahir has worked out the cost of one night and then multiplied by 11.

Slide 7 Rather than working along the number lines, this method looks at the relationship between the lines and gives the rate of £25 per night, a multiplier that is true for all corresponding points on the number lines.

Slide 8 Danny has worked out that 2 is multiplied by 5.5 to get to 11. He has multiplied £50 by 5.5.

Slide 9 Having looked at each of the four approaches in detail, ask students which they found the most efficient. Have they seen an approach that they prefer to the method they used? If so, why do they prefer it? Did anyone use a different approach (e.g. 2 + 2 + 2 + 2 + 2 +1 => £50 + £50 + £50 + £50 + £50 + £25)

Slide 10 Ask students which approach shows the rate per night. Establish that the rate per night is the multiplier between the lines. Although finding the cost of a 1-night stay (as part of Bahir's approach) exposes the corresponding value of £25 on the bottom number line, it is Catia's approach that shows the multiplier between the lines.

Slide 11 Emphasise the proportional relationship between the number of days and the cost. Point out the constant of proportionality in Catia's approach noting that in this approach the arrows show the relationship between the two quantities (the arrow goes between the lines).

Explore 1

Aim	Students work on a multi-step problem involving currencies
Materials	'Cost of stay' handout, Calculators
Slides	Slide 12
Time	15 minutes

In this section of the lesson, students work in pairs to answer questions that examine a number of different relationships:

- number of nights' stay and cost of stay in Euros
- number of nights' stay and cost of stay in South African Rand
- Euros to South African Rand
- South African Rand to Euros

The questions in the task have been carefully sequenced to guide students to complete the table in a way that explores different relationships in a sequential way.

As students work through the task, they are required to share their thinking in pairs, and agree an approach that makes sense to them. This encourages a **collaborative culture** where different **student approaches are valued and built upon** (Key Principles 1 and 5).

What the students might do and what you might do

Ask students to work in pairs and give each pair a copy of the 'Cost of stay' handout. You might like to print the A4 handout and stick it in the middle of a sheet of A3 to give the students some 'working space' around the handout and encourage collaborative working. It is important that students answer the questions in order because they have been carefully designed to guide them to complete the table in a way that explores different relationships in a sequential way. Ask them not to be tempted just to fill in the table.

Make sure that students explain their approach to their partner, working together to agree a solution and develop their own understanding. If you notice that one student is answering all the questions, or they are not working collaboratively, ask a student to explain a question that has been answered by their partner.

Observe students as they complete this task but do not intervene unless necessary. As students work on the task, notice how they illustrate their strategy. As you observe them, identify those with different approaches whom you can call on to explain in the next part of the lesson.

Discuss 2

Aim	To identify different multiplicative relationships
Materials	Whiteboard
Slides	Slide 13
Time	10 minutes

Discuss the task completed in the previous section of the lesson. The focus of this discussion is not on whether students have arrived at the correct answers to the questions, but rather on the approaches and the relationships used to answer the questions. Asking students with different approaches to a question to share their thinking gives value to their strategies and encourages **a collaborative community in which everyone believes everyone can succeed** (Key Principle 5).

What the students might do and what you might do

Ask pairs of students (whom you will have identified earlier) with different approaches to share their approaches to the task. Reinforce their explanations using double number lines on the board or you could use those provided on the slide.

Students are likely to want to check that they have answered the questions correctly but encourage them also to share their approaches to the task.

Answers

- 5 nights: 150 Euros, 11 nights: 330 Euros
- 2 nights: 960 South African Rand, 11 nights: 5280 South African Rand
- 60 Euros = 960 South African Rand, 2400 South African Rand = 150 Euros, 330 Euros = 5280 South African Rand

Explore 2

Aim	Students use a multiplicative structure to explore different ways to solve a proportion problem
Materials	Whiteboard, Mini whiteboards
Slides	Slides 14–16
Time	10 minutes

When completing the ‘Cost of stay’ handout, students had the opportunity to examine a number of different relationships. In this section of the lesson, the relationship between two currencies is explored further and the context of hotel stays is revisited to deepen students’ understanding of multiplicative structure.

What the students might do and what you might do

Slide 14 Establish that the relationship between two currencies is called an exchange rate and is a directly proportional relationship. Emphasise that when one quantity is zero, so is the other. Check that students know that to find how many ZAR in 3 Euros, you need to multiply 16 by 3 (1 × 3 = 3, 16 × 3 = 48).

Slide 15 Depending on the class, you may want students to continue working in pairs or to work on these questions individually. Either way, they should answer these questions on their mini whiteboards. Once the students have had an opportunity to answer the question, use the diagram to explore their thinking. It is likely that there will be a variety of approaches to the task and it is important not to promote a particular method with students.

If students are struggling with the third question, you may need to remind them to look back at how much per night in Euros the hotel cost. They should then be able to see that 60 Euros is the cost of a 2-night stay, so the equivalent in East Caribbean Dollars (192) gives the cost of a 2-night stay in East Caribbean Dollars.

Answers:

- 60 Euros are worth 192 East Caribbean Dollars.
- 1344 East Caribbean Dollars are worth 420 Euros.
- 1344 East Caribbean Dollars is the amount you would pay for a 14-night stay at the hotel.

Slide 16 Discuss ways that a 5-night stay and an 11-night stay can be found.

Answers:

- A 5-night stay costs 480 East Caribbean Dollars
- An 11-night stay costs 1056 East Caribbean Dollars

Explore 3

Aim	To apply understanding to a different context
Materials	Mini whiteboards
Slides	Slide 17
Time	10 minutes

In this section of the lesson, students work on a problem involving distance and time. A double number line is used again to illustrate and explain student thinking. This **promotes coherence** (Key Principle 3) and supports students in seeing that the same mathematical structures apply in different contexts. Covering key content in depth to attain **fluency and understanding** that can be applied in different contexts is preferable to superficial coverage of a larger amount of material (Key Principle 4).

What the students might do and what you might do

Note that it has been assumed that both trains travel at the same average speed.

Depending on the class, you may want students to continue working in pairs or to work on these questions individually. Either way, they should answer these questions on their mini whiteboards. Give students time to work out the distance travelled, and take note of the different approaches that students use. Students may draw their own double number line to illustrate their thinking.

Once most students have answered the problem, discuss the different approaches that students used, using the double number line to illustrate their thinking.

Answer:

- Fi travels 160 km on the train.

Deepening understanding Ask students what the rate of 40 kilometres per hour represents. Can they explain why the multiplier gives the speed of the train?

Review

Aim	To identify the constant of proportionality
Materials	Mini whiteboards
Slides	Slide 18
Time	10 minutes

What the students might do and what you might do

Close the lesson by summarising the different relationships explored during the lesson. Emphasise that the multiplier in a proportional relationship provides all the information you need to calculate all other values.

Tell students that the relationships shown on the slide are all proportional relationships: there is a multiplier that gets us from each value to its corresponding value.
Highlight what the multiplier represents in terms of the different contexts.

- 25 is the rate per night in British Pounds
- 30 is the rate per night in Euros
- 480 is the rate per night in South African Rand
- 16 is how many South African Rand you get per Euro
- 3.2 is how many East Caribbean Dollars you get per Euro
- 40 is the distance travelled per hour (speed)

Practice question

Aim	Students apply their knowledge to an unfamiliar task
Materials	‘Practice question’ handout. It is not necessary to print this out: the question can be displayed on the board or read out.
Slides	Slide 19
Time	10 minutes

What the students might do and what you might do

Students who have previously used an additive approach when solving proportion problems may recognise that finding the cost of 1 kg of apples and adding it on to the cost of 2.5 kg of apples is not the most efficient approach. They may choose to draw a double number line to help in identifying the multiplier.

Ask students whether they have used a different approach to that used prior to the lesson when solving proportion problems. How has their thinking changed? What have they learned about multiplicative structure?