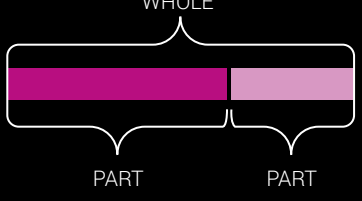
EDUCATION & TRAINING FOUNDATION

CENTRES FOR EXCELLENCE IN MATHS

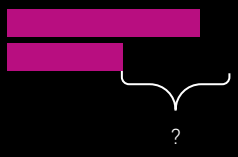
BAR MODELS

AN INTERVENTION FOR FE MATHS PRACTICAL CLASSROOM BOOKLET

PART-WHOLE MODEL



COMPARISON MODEL



Contents

Please scan this QR code to access resources on the Padlet referred to throughout this booklet.

Thank you for doing your bit on behalf of the whole FE maths sector

By doing this intervention you are contributing to the FE Sector's knowledge of what works well in the unique context of FE maths resits as well as improving your learners' experience and progress in FE maths by taking this opportunity to reflect on and develop your own practice.

Summary

This booklet is a practical guide to help plan, prepare and have in your classrooms while trying out a six-week intervention. This intervention was first developed by a group of GCSE maths resit teachers in Further Education colleges in England through action research projects from 2020 to 2022. Then, a larger number of maths resit teachers and learners tried out and evaluated the intervention, to inform the wider FE maths sector and possible future DfE initiatives for FE maths, nationally.

The booklet describes an intervention which may mean introducing something new into your usual teaching practice. Like other teachers who've already tried this intervention, you can integrate it into your existing planning, and make and share adaptations that suit your learners. The intervention is made up of a coherent series of short and focused teaching and learning activities and approaches over a six week period:

Week 1 and 6 are when learners' attitudes and performance is measured using questions provided.

Weeks 2 to 5 are a set of short activities and/or approaches to do with your GCSE maths resit learners.

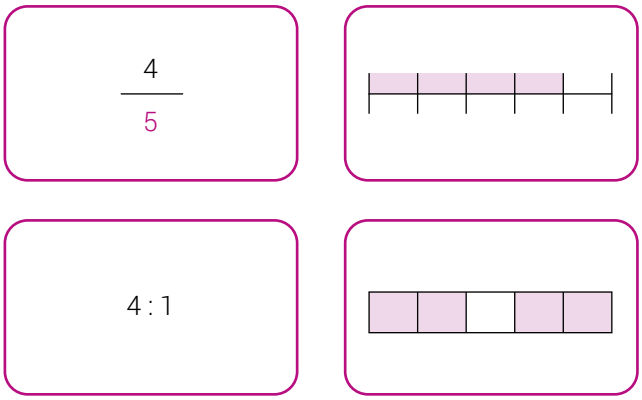
Overview of the intervention

The intervention is an approach to using Bar Models. The following pages summarise what you could do each week and has links for the necessary resources. The intervention was designed for 16 to 19 year old maths resit learners and face-to-face delivery.

Rationale

Many maths learners in FE arrive with prior knowledge and entrenched misconceptions. Some of these misconceptions will persist or return after you teach the content again. To unpick any mathematical misconceptions, it has proved quite difficult to take students back to the concrete phase for mixed ability classes. However, the iconic or pictorial stage in which students create physical and mental images can be supported by the use of models and representations. Bar models encourage students to draw, sketch and doodle to help represent worded questions, whilst giving structure to the teacher to use targeted questioning to help students explore their thinking and uncover misconceptions.

For this intervention we are focusing on three core topics – Fractions, Proportion and Ratios – as these are key to success in GCSE (and Functional Skills) qualifications and are usually taught in the Autumn term. The intervention can be used as a new way for learners to access a range of GCSE topics and as ongoing learning checks while you teach these three topics for the first time. It can also be used for retrieval and interleaving of these topics after you have taught them. Importantly, the intervention includes time to check what students have learned and to address remaining misconceptions directly.



Evidence for the intervention idea

Building on from the very successful Singaporean approach of using bar models in schools, recent research done by groups of FE maths teachers in a small number of colleges as part of the Centres for Excellence in Mathematics (CfEM) programme, and from South East Asia and the schools sector, suggests that using the bar model approach to teaching could help learners embed conceptual understanding and visualise and contextualise all relevant information in a question. This is especially important for complex word problems where learners might get confused by the 'surface' information. Findings from this research have fed into a practical approach that is ready to be tried out by more teachers.

Introduction to the intervention

Access the Bar Models intervention Padlet (scan QR code on page 2 to access) to watch/rewatch recordings of two CPD sessions. These show how to use bar models as a tool for learning through a series short activities leading up an exam question focus in the final week. There is also a short Bar Modelling Intervention induction video.

Timeline and support

Access the Bar Models intervention Padlet (scan QR code on page 2 to access) to watch/rewatch recordings of two CPD sessions. These show how to use bar models as a tool for learning through a series of short activities leading up an exam question focus in the final week. There is also a short Bar Modelling Intervention induction video.

The research that took place in Autumn 2022 followed the timetable below, though teachers doing this intervention after this date can do it at any suitable time of year.

Autumn term	Half term 1	A. Induction	• Watch introductory **Bar Models intervention video** (Available on the Padlet). • Planning discussion with colleague(s).
		B. Training	• **CPD:** Re/watch Session 1. (Available on the Padlet).
	Half term 2	C. Intervention	• **Week 1** Baseline measures. • **Week 2** Introduce bar models through examples and questions on fraction of amounts. Get students familiar with drawing and using bar models to represent questions visually (40 mins). • **Week 3** Introduce "part" and "whole" – finding the whole given a fraction. Practice drawing bar models to represent the question visually (40 mins). • **Week 4** Try conceptual variation and multiple representations in an engaging card sort (20 mins) • **Week 5** Develop fluency in using the skills developed over the previous weeks to divide by a ratio (20 mins). • **Week 6** Follow-up measures.
Spring term	Half term 1	D. Reflection	• **Reflect on what worked well and you'll continue. Share with colleagues. For example, in January.**

If doing the intervention in the Autumn term, your **Week 1** is likely to be in the same week that November resits end or the one after that. Ensure you print enough paper maths assessments and questionnaires to give to each of your learners, and the mark scheme – all available on the Padlet. Create unique anonymising codes for the front cover of the assessment and questionnaire only if wanted.

Then, in **Weeks 2 to 5**, you will be able to show learners how they could use bar models to represent worded problems and help to address some of their misconceptions. There is weekly reflection box which can be used to record notes on what you did and how it went.

In **Week 6**, the same learners (or as many as attend) do the follow-up measures, so that results can be compared with the baseline. Again, this must be done class time to ensure independence, pastoral support and high completion rates.

Note: Teachers taking part in the research in Autumn 2022 received an online questionnaire to feedback their experiences and views on the effectiveness of the intervention. This gave invaluable insights on how well the intervention worked and how it could be improved. Independent researchers published anonymised and aggregated findings in March 2023.

Week 1

Baseline measures: maths assessment and attitudinal questionnaire

Introduction

By now, you'll be aware that there are two baseline measures for all your 16-19 year old maths resit learners and that these are available on the padlet along with all other intervention resources:

1st a paper-based maths assessment, with approximately 10 GCSE-type questions of varying difficulty on fractions, ratio and proportion.

2nd a short questionnaire that mainly focuses on their attitudes to learning maths.

Before starting, plan when in Week 1 to do both these baseline measures. In Week 1, please give learners the maths assessment first and the attitudinal questionnaire second. This is because there is a question in the attitudinal questionnaire that asks about their experience of doing the maths assessment. Both baseline measures can be used as learning opportunities.

Note that the same approach to data collection is used in Weeks 1 and 6.

Activity

Inform all learners about the research (10 mins). For example, you could say that you are taking part in some research which involves trying out a new approach in your teaching. You'd really like their help! The aim is to find out how well that approach works so that I know whether I should use it again with other learners in future. Tell your learners that they have the right to opt out of participating in this research at any time and be ready to provide alternative activities just in case.

Baseline maths assessment (30 mins). After informing all your learners about the research and giving out the unique identifier codes, please ask them to complete the maths assessment. Papers are expected to take approximately 30 minutes (though some may need a little extra time). Review the scripts by looking for methods and mis-concepts in workings out and skipped questions as well as marks.

Baseline attitudinal questionnaire (10 mins). Download a print-ready version of the questionnaire from the padlet. Please distribute a printed copy to all your learners during a maths lesson and explain that this is to find out their thinking about maths (it isn't a test). To re-iterate, the questionnaire should be completed by learners in class, face-to-face with you. This is to motivate and support learners to complete the task, independently. Please do not ask learners to do the questionnaire in their own time or remotely.

Supervise their completion of the questionnaire. Each individual learner should complete their own questionnaire. The questionnaire has approximately 15 questions. It is expected to take 10 minutes but there is no strict time limit. The questions are about the learners' confidence and experience of learning maths, in general. Teachers must not answer questions for learners or suggest answers but can help with question comprehension. Strict exam conditions are not required.

Week 2

Introduction to bar models (40mins)

Introduction

This week's task is designed for all learners, whether new to bar models and or with a little more experience. As the questions become more challenging over the 4 weeks they will have benefited from "joining in" from the start as they will have become familiar with drawing a bar model to help structure their thoughts and answer questions they would have previously missed out in their exam.

Resources

- Intervention PowerPoint – Activity 1 available on the Padlet (scan QR code on page 2 to access).
- Suggestion: 1cm square paper for students who may struggle to draw the bar models appropriately OR mini whiteboards/pens (easy for students to correct drawings).

Activity

Go through the PowerPoint slides (see link in Resources above).

At the beginning of the slides is a baguette representing a bar and questions asking your group to find fractions of amounts including those greater than 1.

The numbers chosen in the questions have been designed to be accessible but also useful for the bar model representation but feel free to and adjust the numbers to reflect your cohort.

Realistically, in this first task, you may find some learners are reticent to try something different. So this lesson requires a conciliatory tone and skill to manage these situations.

An important part of the approach is not to discount their existing strategies but to use bar models as a support for the explanation of their methods. For example, you might say "Using a bar model, show me why ¾ of 20 is 15."

Week 3

Finding the whole given a fraction (40 mins)

Introduction

In Week 3, you're going to build on your students' understandings of how to draw a bar model and how they can visually represent a question using a bar model. Working in reverse to the last activity (fraction of amounts), students will look to find the whole given a fraction, using the bars and braces model.

Resources

- Intervention Power Point – Activity 2 on the Padlet (scan QR code on page 2 to access).
- Suggestion: 1cm square paper for learners who may struggle to draw the bar models appropriately OR mini whiteboards/pens (easier for learners to correct drawings).

Activity

10 mins - Introduce this task by asking learners to model the **35 cubes question** on the board using a bar model. Explain the use of the bar and braces after learners have had time to draw their initial ideas. You might ask:

- "How many "parts" represents 35 cubes?"
- "Is each part equal?"
- "What is the question asking us to find?"
- "How do we label this on the bar?".

Try to use their suggestions initially to guide the whole class towards the model used on the slide.

Then, reveal the model on the slide after a class discussion. Through some direct questioning, label each "part" as 35 to help students visualise why the whole would equal 140 cubes.

After each slide, allow learners time to ask any questions before moving on.

10 mins - **The frisbee activity** is on the next section of the slides. It can be completed individually by learners or in small groups – whichever the teacher prefers. The idea is for students to practice drawing bar models and labelling them correctly with braces. Whiteboards and pens could be useful here. Talk through each bar model using the responses from the group, as before.

20 mins - Use the last three slides of the same PowerPoint for learners to further develop their **understanding of fractions** with the use of bar models. If learners are reverting back to their preferred method for these GCSE style questions, allow them to but prompt them for explanations. If they are unable to provide an explanation of how they arrived at their answers, suggest that a bar model may help them to do so.

If students want to find out more, you could direct them to further reading and resources on using bar models at BBC Bitesize: solving problems using bar models.

Week 4

Ratio notation and interpreting ratio (30 mins)

Introduction

In Week 4, you'll introduce learners to conceptual variation and multiple representations in an engaging card sort. This activity bridges the learning and understanding gained in Week 2 and 3 so that learners will be able to explore bar models with ratios in Week 5.

Resources

- Intervention Power Point – see Activity 3 on slides on the Padlet (scan QR code on page 2 to access)
- Print and cut Card Sort Activity (A3 for a better print), for pairs or small groups no larger than 3 to work on first. Available on the same Padlet.
- Interactive Card Sort Activity using Desmos software, for your whole group to focus on.

Activity

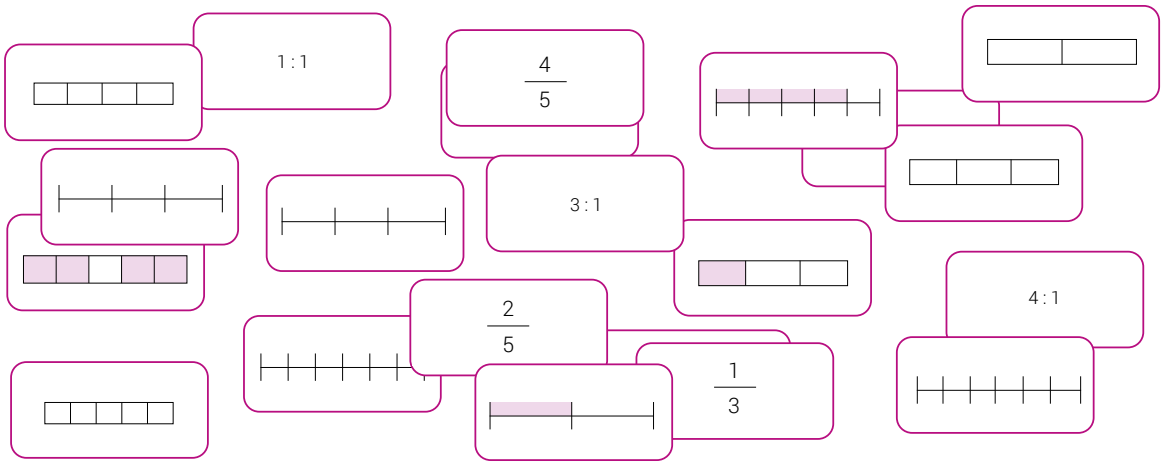
Ask learners to work in pairs (ideally) and to sort the cards into groups of four where each model is representing the same quantities. Learners should be warned that some information from the cards is missing. Also, if doing it on paper, completing using a pencil first will allow for any mistakes to be corrected, if necessary. Encourage discussion in the pairs (or 3s).

Circulate the room to check the groups made by each pair/group, asking prompting questions such as:

- Can you explain why 1:2 is in the group matched with 1/3?
- What similarities do notice with the bar model and shaded number lines?
- If the shaded part was equal to 20kg, what would the whole represent?

The activity is available online using a free and easily accessible software called Desmos.
Share it, projected, so that you can have whole class discussions on which cards were matched together:
https://teacher.desmos.com/activitybuilder/custom/632064cec185f8060659ec45

Match each bar model to a ratio. Match each number line to a fraction. Some cards have information missing for you to complete.



Week 5

Ratio with bar models (40 mins)

Introduction

In Week 5 of delivering this intervention, learners will look in-depth at contextual ratio exam-style questions.

Resources

- Intervention PowerPoint – see slides called 'Activity 4' on the Padlet (scan QR code on page 2 to access).
- 2x ratio worksheet – print double sided (hint sheet should be on the reverse of blank answered questions) from the same Padlet.

Activity

The activity begins with 'Activity 4' on the PowerPoint slides that show the sharing of money between Jamal and Harry. Each of the three questions on the slides are similar in their wording but the proportion of money given varies (total amount given between the pair, part amount given or a difference given). This variation gives scope for the teacher to explore key ideas and concepts with their class.

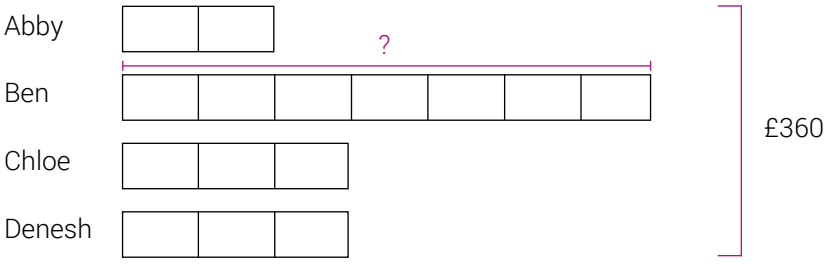
It is important for the teacher to draw/explain the bar models with the class, in order for learners to notice the difference between the labelling of each question from the information given. Teachers should try to emphasise these differences as they walk through the three examples, which noticing how the ratios are intentionally remaining the same throughout.

The next part of the activity is designed for students to take the examples just explored whilst practicing some conceptual exam-style questions themselves. Six questions are available on the worksheet. The worksheet should be printed double sided with hints on the back to support learners who may need it. Encourage learners to try to draw the bar models and braces with labels using the information given in the question.

A mini-plenary consisting of a GCSE Past Paper question on Ratios is to be used at the end of this

Exam Question

£360 is shared between Abby, Ben, Chloe and Denesh.
The ratio of the amount Abby gets to the amount Ben gets is 2:?
Chloe and Denesh each get 1.5 times the amount Abby gets.
Work out the amount of money Ben gets.



Week 6

Follow-on measures: maths assessment and attitudinal questionnaire

Introduction

In Week 6, please repeat the process that you followed in Week 1.

Activity

In Week 6, please repeat the process that you followed in Week 1 so that all – or as many as possible – of your 16-19 year old maths learners complete both measures:

1st a paper-based maths assessment, with approximately 10 GCSE-type questions of varying difficulty on fractions, ratio and proportion.

2nd a questionnaire that mainly focuses on their attitudes to learning maths.

Once learners have completed the assessment and questionnaire, review your weekly reflections and compare the results from Week 1 and Week 6.

Conclude your research by deciding what to teach particular learners next and what worked well enough to continue or adapt in future and share with colleagues.

Key points:

- In Week 6, please prepare for and administer both measures – the follow-on maths assessment paper (30 minutes) and then the questionnaire (10 minutes). Return to Week 1 (page 4) for details.
- Both measures do not have to be done immediately after each other or in the same lesson, but they do need to be completed in Week 6.
- Compare results from Week 1 and 6, and reflect again on Weeks 2 to 5. Use this evidence to inform your future teaching.

Thank you

If you enjoyed this intervention and want to find out more, you can find further reading and resources on using bar models on the following links: BBC Bitesize – How to use the bar model to solve problems www.bbc.co.uk/bitesize/topics/znmtsbk/articles/zfv6pbk.

Teacher reflection/notes

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