Lesson plan

Frequency charts and averages

Contents

1. Rationale

When working with frequency charts, students often struggle to see the relationship between data presented as a list and the same data represented on a chart. The use of sticky notes in this lesson, to both record and display data values provided by students, helps students to identify their own data values within a frequency chart and develop their understanding of the relationships between different representations of data. **Using multiple representations provides insight into the mathematical structure** of data sets and exposes the way in which the three averages provide a summary (Key Principle 1). **Developing both fluency and understanding** is an important part of the mastery approach (Key Principle 4) and in this lesson, time is spent interpreting and comparing various data sets represented using frequency charts and summary statistics. By exploring these different representations, students are supported in developing a deeper understanding of the way in which the mode, median and mean represent the average of a set of data and the distinction between measures of average and range as a measure of spread.

2. GCSE curriculum

Statistics

S2 interpret and construct tables, charts and diagrams, including frequency tables

S4 interpret, analyse and compare the distributions of data sets … through measures of central tendency (median, mean, mode and modal class) and spread (range)

3. Lesson objectives

- Construct and interpret frequency charts
- Find the averages and range for a set of data
- Understand how the mean, median, mode and range summarise data
- Use different representations of data to provide insight into mathematical structure

4. Starting points

This lesson assumes that students have some experience of using an average value to summarise a data set. It is likely that they will be most familiar with the mean as a measure of average.

5. Research questions

Pedagogic focus
In which ways does the lesson develop the students' understanding and fluency related to averages?

Maths focus
How do students use different representations of data to develop their understanding?

6. Lesson structure

Activity	Time (min)	Description/Prompt	Materials
Introduction	5	Introduce the context of exercising to keep healthy and survey the class on how active they have been in the last 5 days.	Mini sticky notes, flip chart paper or 'Class results' handout Slides 2 and 3
Explore 1	10	Tell students that Yaima runs two fitness classes. She surveys the attendees of the first class and uses the results to construct a frequency chart. Explore the mode, median and mean as measures of average.	Mini whiteboards Calculators Slides 4–13
Explore 2	5	Tell students that Yaima surveys her second fitness class and represents the results in a frequency table. Give students a couple of minutes to engage with the data.	Mini whiteboards Slide 14
Discuss 1	15	Tell students that Yaima's colleagues Abi, Baz and Chloe discuss the data and make some incorrect statements about the survey results. Ask students to identify why the statements are not true, Highlight the distinction between averages and the range, which is a measure of spread.	Slides 15–18
Explore 3	25	Introduce the main activity and ask students to work in pairs to match the frequency charts and tables to the corresponding alternative data representation.	'Representing data' handout, 'Cards' and scissors Calculators Slides 19 and 20
Review	15	Check that students have matched the cards correctly. Bring the learning together and discuss specific features of the data.	Mini whiteboards Calculators Slides 21–27
Practice questions/ Discuss 2	15	Ask students to complete two practice questions and discuss their thinking.	'Practice questions' handout Calculators Slides 28 and 29

7. Teacher guidance

Introduction

Aim	To develop students' understanding of the relationships between different representations of data
Materials	Mini sticky notes, flip chart paper/ 'Class results' handout
Slides	Slides 2 and 3
Time	5 minutes

At the start of the lesson, students use sticky notes to construct a frequency chart showing how active the class has been in the last five days. All students are actively involved and can see where their own data value features in the chart. By asking all students to make a contribution and comment on the results, a **collaborative culture** is encouraged (Key Principle 5).

What the students might do and what you might do

Slide 2 Tell students that one of the things that we are advised to do to keep healthy is to exercise/be active. Discuss some different ways that we can be active, such as going for a walk/walking to college, playing sport, dancing, riding a bike, and so on.

Give each student a sticky note and ask them to write down the number of days out of the last five days that they did something physically active for at least 10 minutes (a whole number between 0 and 5), and to put their initials in the bottom right corner.

Ask students to stick their sticky notes at the front of the class and then tell them that we are going to display the results on a class frequency chart.

Slide 3 Display the axes (using flipchart paper, the 'Class results' handout enlarged onto A3 paper, or Slide 3) and ask students to place their sticky notes in the frequency chart.

Ask students to comment on the chart. Encourage *all* students to contribute something that they notice to the discussion. You may like to ask students some specific questions about the chart to initiate discussion. For example, '*How many people in the class were active on all five days?*' or, '*What did most people do?*'

Students may not comment on any statistical measures, and it is not necessary to introduce these at this stage. Being able to identify their own sticky note and as a result recognise where their data value is represented in the chart can play an important part in developing their understanding of the relationships between different representations of data.

Explore 1

Aim	To explore the relationship between different representations of data
Materials	Mini whiteboards, calculators

Slides	Slides 4–13
Time	10 minutes

In this section of the lesson, students explore the three averages and how they each provide a summary of the data. Statements are provided for students to discuss, giving them a starting point and helping to expose common misconceptions so that **students' existing knowledge can be identified and built upon** (Key Principle 2).

What the students might do and what you might do

Slide 4 Tell students that Yaima runs two fitness classes on a Saturday morning. One class is at 10 am and the other class is at 11 am. She is thinking about running a new class during the week.

Slide 5 Yaima wants to know how much additional exercise the attendees of her class already do. At the end of the 10 am class she asks the 11 people who attended to write on a slip of paper, the number of days out of the last 5 days they have exercised for at least 30 minutes.

Slide 6 Tell students that Yaima puts the results into a chart. You should allow students enough time to see how she does this. She shows it to her colleagues Abi, Baz and Chloe. Ask students for some observations about the data. Then ask them what they think the advantages are of the chart over the list of numbers.

Slide 7 Abi, Baz and Chloe start to discuss how to interpret the data and Abi asks what the chart shows. Ask a couple of students to comment on the chart. They may, for example, say that no one in the class exercised on just one day or that most of them exercised every day for the last five days. If students refer to the mean, median or mode, discuss these using the next slide.

Slide 8 Tell students that Chloe suggests finding the average. Ask them what Chloe means by this and assess their understanding of how an average value provides a summary of a data set. Students may refer to the mean, median and mode during the discussion. These average values will be explored further as part of the discussion of Abi's, Baz's and Chloe's statements in the slides that follow.

Slide 9 Abi says, 'The average number of days is 3.'
Baz says, 'The average number of days is 5.'
Chloe says, 'The average number of days is 4.'

Ask the students how all three averages can be correct. Provide calculators and ask students to discuss in pairs what each of Yaima's colleagues has done. If they struggle, suggest counting the number of items and putting them in order. At this stage, encourage them to work in ways that are familiar to them in order to access the data.

Slide 10 Students' initial thoughts on averages may be to calculate the mean, as this is the most frequently used measure of average. This is what Abi has done.

Discussing with students how many people attended Yaima's class is important in establishing the total number of values in the data set. Students often understand the calculation of the mean as involving the need to divide by '*the number of things that there are*', which can sometimes be misinterpreted when the data is represented in

frequency tables/charts, rather than in a list. Check that students can identify the 11 attendees from the chart as well as from the list of responses.

Slide 11 Ask students to explain to the class what Baz has done. Establish that Baz has identified the mode, which is the most common response. 4 of the 11 people in the fitness class exercised for at least 30 minutes on all 5 days.

Slide 12 Students can sometimes mistakenly identify the mode as the frequency of the value, rather than the value itself. It is important to check whether students have correctly identified that Chloe has found the median value or whether they are incorrectly interpreting 'the average number of days is 4' as the mode, mistaking the highest frequency (4) as the mode rather than the number of days represented (5). Ask students to explain why Yaima may think that Chloe has found the mode and establish that Chloe has identified the median (6th value).

Slide 13 Summarise the definitions of the three averages and highlight why we consider the different measures of average rather than just one. You might like to discuss how the mean is calculated using all the data values, but may not be a good measure of average if the data contains outliers. The median is not affected by extreme values and can provide a better measure of the average than the mean when the data is skewed. The mode is the least used measure of average, but it is the only measure that can be used for nominal as well as numerical data.

You may like to write the definitions on the board so that students can refer to them during the lesson.

Explore 2

Aim	Students identify features of data represented in a frequency chart
Materials	Mini whiteboards
Slides	Slide 14
Time	5 minutes

In this part of the lesson students are given the opportunity to apply their understanding to a new data set. They interpret results from a different survey represented in a frequency table.

What the students might do and what you might do

Slide 14 Tell students that Yaima asks the same question to the 12 people attending her 11 am fitness class. She represents the results in a frequency table for this class and shows it to Abi, Baz and Chloe. Give students a couple of minutes to engage with the data. They may want to note down on their mini whiteboards what they can identify from the data. It is not necessary for them to work out the correct values for the mode (4), median (3.5) and range (3), but some may want to do so.

Tell students that Abi, Baz and Chloe are discussing the data. Slides 15-17 explore what Abi, Baz and Chloe say.

Discuss 1

Aim	To explore common misconceptions
Slides	Slides 15–18
Time	15 minutes

The three statements are discussed here. The focus of the discussion is on the errors made, as well as establishing the correct values for the mode, median and range. During the discussion, observe what is revealed about **students' existing misconceptions** (Key Principle 2).

What the students might do and what you might do

Slide 15 Ask students what mistake Abi has made and what the correct answer is.

She has made the mistake of identifying the frequency of the most common result as the mode, rather than the number of days.

Slide 16 When discussing Baz's statement it is important to discuss how to determine the median when there is an even number of values in the data set. It is also important for students to recognise that unlike the mode, the median (and mean) can be a value that does not correspond to an actual event. In this case, the median is 3.5 days, even though a person cannot actually have exercised for at least 30 minutes on 3.5 days.

The total frequency (i.e. the number of fitness class attendees that responded to this survey) is an important value that is needed to establish both the median and the mean. You may want to also discuss how to use a frequency table to determine the mean.

Slide 17 Ask students whether they think Chloe is correct or Yaima is correct. Discuss how Chloe may have thought the range is 5 by considering the difference between the greatest possible number of days and the smallest possible number of days (5 – 0 = 5) in the table while failing to take into account that no one exercised for 0 days or for 5 days. Establish that Yaima is correct (4 – 1 = 3) and highlight to students the distinction between averages (mean, median and mode) and range, which measures the spread of the data.

Slide 18 Before moving on to the next activity, spend a couple of minutes comparing the results from the two classes. Displaying the results for the second class on a frequency chart should help students make a direct comparison. Discuss the advantages of the frequency chart over the frequency table and ask students to explain how they identify the range and average values using the chart.

The summary statistics suggest that the attendees of the first class are exercising slightly more often. Ask students to explain how this can be identified from the charts. Highlight that while the attendees of the first class seem to be exercising more often, some of them have not exercised at all in the last 5 days, unlike for the second class where everyone has exercised.

Explore 3

Aim	To match frequency charts and summary statistics that represent the same data
Materials	'Representing data' handout, 'Cards', scissors (if the cards are not already cut up), calculators
Slides	Slides 19 and 20
Time	25 minutes

Before students begin the task, the process of matching charts and summary statistics tables is modelled. During the task students are required to perform repeated average calculations, providing an opportunity for them to develop both their **fluency and a deeper understanding** (Key Principle 4) whilst making **connections between different representations of the data** (Key Principle 3).

What the students might do and what you might do

Slide 19 Tell students that the two tables give a summary of the data represented in the two charts. Ask students which table goes with which chart and how they know. Emphasise that the mode is the easiest average to identify from the chart.

Once the tables/charts have been correctly matched, ask students to complete the missing values in the two tables. Encourage students to use the chart directly when calculating the mean. Emphasise how to identify the median when there is an even number of data values. Remind students of the importance of the total frequency when finding both the median and the mean.

Slide 20 Tell students that Yaima carried out the same survey during her next eight fitness classes. She recorded the results in charts and made some tables of summary statistics. Distribute to each pair of students a copy of the 'Representing data' handout and the 'Cards'. Ask students to work in pairs to identify which chart goes with each table of summary statistics. Tell students that they need to fill in the missing values in the tables.

Encourage students to take turns when matching the tables and charts. It is important that students **work collaboratively** (Key Principle 5) and can explain their thinking when placing a card.

If you think students might struggle, you might like to give them the first page of the handout for fitness classes 1–4 only and cards C1, C4, T2 and T3. Alternatively they could be given the full set of cards with just the first page of the handout. To help students to get started, you may want to ask them to count the number of people in each class and think about how this might help them.

As students work, observe whether they group the charts and tables cards in any way. They may, for example, group tables T1, T2 and T3 together as they all have a range of 4, whereas the range in table T4 is 5. If so, ask students if they can group the tables in any other way (no, as some of the values for the other statistical measures are missing from the tables and would need to be completed first) and which statistical measure would be most helpful to use if they could. Use the discussion to check whether students recognise which of the four statistical

measures are easiest to identify from the chart and how they might use this to help them match the charts and tables.

Review

Aim	To review the matching task and explore features of the data
Materials	Mini whiteboards; calculators
Slides	Slide 21–27
Time	15 minutes

The task is now reviewed, with an emphasis on checking students' understanding. It is likely that students will have identified the mode and range initially to aid with matching the tables to the charts, as these are easier to determine. For some of the fitness class results, students would need to work out the other statistics to determine the correct match.

What the students might do and what you might do

Slide 21 Once students have had sufficient time to make progress, hold a class discussion. While it is important to establish the correct matches, focus also on bringing the learning together and drawing out comparisons between the different data sets.

Use Slides 22–26 to explore some specific features of the data.

Asking questions such as those suggested below or prompting students using the methods described may be helpful in checking students' understanding.

Slide 22

Bimodal data

The results for fitness classes 2 and 5 contain data that is bimodal.

- Ask students to describe the characteristics of the chart they were looking for when matching a chart to the table for fitness class 2.

Students may say that they looked for a chart where the bars representing 2 and 3 days are the same height. Check students' understanding of modal number of days as most common, so we are interested in charts with two bars of equal height that are taller than all the other bars. Establish that the results for fitness class 5 are also bimodal.

Slide 23

Data that has no mode

The frequencies for fitness class 4 are all 3 (for 0 to 4 days inclusive) so all the bars in the frequency chart are the same height.

- Ask students to explain why the table for fitness class 4 had no modal value completed and how this helped them to identify the matching chart.

Finding the median

After matching the charts and tables for fitness classes 6, 7 and 8, the median number of days needed to be identified. There were 14 attendees for all three classes so there is an even number of data values. Check students' understanding of how to find the median when there is an even number of data values and highlight the differences for each data set.

Slide 25 Conclude the discussion by asking students to compare the charts for fitness classes 1 and 3 and comment on what they notice. They should notice that they are the same, but that the chart for fitness class 3 is shifted one to the right.

Discuss the differences between the mean, median, mode and range, noting that the range remains the same but that the averages in T2 are one more than in T3.

Relate the charts to the amount of exercise reported by the participants of fitness classes 1 and 3; the participants of fitness class 3 exercise one more day than those of fitness class 1.

Slide 26 Bring students' thinking together by summarising the three different averages. Ask students to find the mean, median and mode for the two data lists:

Data List A: 1, 2, 3, 4, 5
Data List B: 1, 2, 3, 4, 4, 4

After a couple of minutes, discuss the three values for the mean, median and mode for the two data lists and highlight the different ways in which they measure the average. You might like to discuss how the mean is calculated using all the data values, unlike the median and mode, and how there can sometimes be no mode (as in the case of the first data list) or more than one mode. Check that students are able to identify the median when there is an even number of data values (as in list B) compared with identifying the middle value as in list A.

Once the values for the three averages have been identified for the two lists, ask students which average they think best describes each of the data lists. Contrast data list A, where the mean and median have the same value, with data list B, where the three values are different. Discuss how the mode may be considered as the best average for list B since half of the data values are 4, or how the median could be thought to be the best average as it takes into account the values that are less than 4. Identify that the mean may not be the best measure of average, especially when the data is skewed as it is here, and reinforce how taking each measure of average separately, without the context of the other two, can sometimes give a misleading summary of the data.

Slide 27 Discuss the range and establish how it uses the extreme values to describe how spread out the data is. Ask students to find the range for the two data lists and establish that it is the same for data lists C and D. Ask students to describe what is different about the two lists.

Ask students to determine the mode, median and mean for the two data lists and write them on their mini whiteboards. Acknowledge that the range sometimes provides an unreliable measure of how variable the data is, as demonstrated here by two sets of data with the same range but different average values.

It is important that students recognise the distinction between measures of average and range as a measure of spread, and how using the range and average values together can provide a more reliable summary of the data.

Practice questions/Discuss 2

Aim	Students apply their knowledge to an unfamiliar task
Materials	'Practice question' handout. It is not necessary to print this out: the questions can be displayed on the board.
Slides	Slides 28 and 29, calculators
Time	15 minutes

In this part of the lesson students apply what they have learned in the lesson to two practice questions.

What the students might do and what you might do

Slide 28 Give each student a copy of the 'Practice questions' handout. Give students a couple of minutes to work on the questions individually and then discuss their thinking.

When completing the first practice question, it is important that students explain what Bridgit has done wrong and not just state that she is incorrect because the median is actually 15. Observe whether they write out the list in order and if so, how they use this to identify the range as being 10 in part (b).

When discussing the first question, establish the importance of ordering the data and how putting data into a frequency table helps with this. You may want to point out that there is no mode for the list of numbers.

Slide 29 For the second practice question, students may just find the total of the frequency column to give an answer of 30 (the number of students). If they do, you may wish to ask them the total number of points scored by the five students who each scored 5 points.

Ask students to describe their method when working out the total number of points scored in part (b) and explain why it works. Discuss how adding an additional column to the table may help with finding the total number of points scored. Establish why working out the total number of points scored is a useful thing to do (because we can use it to calculate the mean).