Lesson plan

Algebraic equations

Contents

1. Rationale

Students are usually familiar with using letters to represent variables but can struggle to understand the mathematical structure of an equation. In this lesson algebraic equations are used to model real-life situations and students explore the effects on the meaning of equations and expressions when the definitions of the variables change.

Developing an **understanding of mathematical structure** (Key Principle 1) through mathematical representations is a key part of the teaching for mastery approach and in this lesson the use of representations is encouraged when exploring relationships.

Possible incorrect equations are discussed, to **expose common misconceptions and provide insight into students' existing knowledge** (Key Principle 2). Providing different possible solutions can give students a starting point, **building confidence and encouraging them to share their understanding** (Key Principle 5).

2. GCSE curriculum

Algebra

Notation, vocabulary and manipulation

A21 translate simple situations or procedures into algebraic expressions or formulae; derive an equation, solve … and interpret the solution

3. Lesson objectives

- Make connections between real-life situations and algebraic equations
- Solve equations and interpret the solutions
- Use representations to provide insight into mathematical structure

4. Starting points

This lesson assumes that students have used letters to represent numbers that can be varied and are familiar with substituting into a formula.

5. Research questions

Pedagogic focus

In which ways does the teacher encourage students to think about mathematical structure?

Maths focus

How do students use representations when working with equations?

6. Lesson structure

Activity	Time (min)	Description/Prompt	Materials
Introduction	20	Introduce the context of selling skincare products and explain to students that Tia's dad has ordered a supply of face wipes to sell online. The face wipes are delivered in multipacks containing six packs of wipes. Tia's dad asks Tia to help with writing an equation that describes the relationship between the number/cost of packs of wipes and multipacks. Ask students to discuss in pairs the equations written by Tia and her friends, Vinay and Sade and check students' understanding of the mathematical structure of the equations.	Mini whiteboards Slides 2–12
Explore/ Discuss 1	10	Tell students that Tia's dad sells 'Basic' and 'Deluxe' wipes and has received some multipacks from the supplier. Information about the multipacks is provided in three equations. Ask students to write down what each of the equations tells us.	Mini whiteboards Slide 13
Explore 2	25	Tell students that Tia's dad has orders from his online shop that are ready to be posted. Ask students to work in pairs to complete the statements/equations and use this information to work out how much it will cost Tia's dad to send the parcels.	'Completing descriptions' handout Slides 14–15
Discuss 2	15	Ask students to explain their thinking and discuss different approaches. Compare students' ways of working to Tia's approach to the task.	Mini whiteboards Slides 16–20
Review	10	Review students' understanding of the mathematical structure of equations.	Mini whiteboards Slides 21–23
Practice questions /Discuss 3	10	Ask students to complete two practice questions and discuss their thinking.	'Practice questions' handout Slides 24–25

7. Teacher guidance

Introduction

Aim	To introduce the context and explore the relationships between single packs and multipacks of wipes
Materials	Mini whiteboards
Slides	Slides 2–12
Time	20 minutes

In this first section of the lesson the relationship between a single pack and a multipack containing six packs of wipes is explored. Students compare the **mathematical structure** of equations that represent the number of packs of wipes with those for the cost of the wipes. They are encouraged to consider pictorial representations of the relationships to help to reveal the **mathematical structure** (Key Principle 1).

What the students might do and what you might do

Slide 2 Tell students that Tia's dad sells skincare products online and has just ordered a supply of face wipes.

Slide 3 The face wipes are delivered to Tia's dad in multipacks containing six packs of wipes.

Slide 4 Tia's dad wants to be able to determine how many packs of wipes he has received, from the number of multipacks that have been delivered. He asks Tia if she can write an equation that describes the relationship between the number of multipacks and the number of packs of wipes. Tia asks her friends Vinay and Sade to help her.

Give each student a mini whiteboard and ask them to write an equation for the relationship between the number of multipacks and the number of packs of wipes, using p to represent the number of packs and m to represent the number of multipacks.

You might like to ask students to think about which is bigger: the number of multipacks or the number of packs. Students may also need reminding about the rules for writing algebra, for example, not including the multiplication sign (i.e. $a \times b$ is written as ab).

After a couple of minutes, ask students to show you their mini whiteboards.

Slide 5 Show students Tia's equation ($m = 6p$). Check whether any students have the same equation as Tia. Tell students that Vinay doesn't think that Tia's equation is correct and has a different equation ($p = 6m$).

Survey the class to see if any students have written down the same equations as Tia and Vinay. You might like to ask students with the same equations as Tia and Vinay to explain their thinking.

Use Slides 6-8 to support a discussion of the relationship between the number of multipacks and the number of packs of wipes.

Slide 6 Students may wrongly assume that because the number of multipacks is known, *m* should be the subject of the equation as in Tia's equation. They may read Tia's equation as 'a multipack equals 6 packs of wipes' and conclude that Tia's equation is therefore correct. If this is the case, suggest that students substitute some numbers for *m* and *p* to check whether Tia's equation works. For example, If Tia's dad was sent **2 multipacks** of wipes, he has a total of **12 packs** of wipes. 2 x 6 = 12, so you need to multiply the number of **multipacks** by 6 to get the number of **packs.** Remind the students that the number of packs is always bigger than the number of multipacks.

Slide 7 Check that students understand why Vinay's equation is correct and recognise that multiplying the number of multipacks by six gives the total number of packs of wipes.

Slide 8 Introduce Sade's equation and ask students to think about how it represents the same relationship but in a different way (she has written the number of multipacks as the subject rather than the number of packs of wipes).

Writing Sade's equation with *p* as the subject rather than *m* gives Vinay's equation, which is the equation that Tia's dad needs.

Slide 9 Using a pictorial representation of the relationship between *p* and *m* may be helpful when exploring the relationship between Vinay's and Sade's equations.

Slide 10 Tell students that Tia's dad decides that he is going to offer the face wipes for sale online, either as a single pack or as a multipack containing 6 packs of wipes (since they arrive already packaged in multipacks of 6 wipes). He knows the unit cost of a pack of wipes and wants to work out the cost of a multipack of 6 packs. He asks Tia if she can write another equation to describe the relationship between the cost of a pack of wipes and the cost of a multipack. Tia asks her friends for help again.

Ask students to suggest an equation and to write it on their mini whiteboard before going on to look at Tia's, Sade's and Vinay's suggestions on the next slide.

Slide 11 Show students Tia's, Vinay's and Sade's equations. Ask students whose equation they think correctly describes the relationship between the cost of a pack of wipes and the cost of a multipack. Remind them to think about which is the bigger number, *x* or *y.* Ask them to write the initial of the person they think is right on their mini whiteboard and then show it to you.

If there is agreement about who is right, ask a couple of different students to explain their thinking. If there is disagreement, ask students with different views to explain their thinking.

You can use Slide 12 to support a discussion of why Tia's equation is correct.

Slide 12 Check that students understand and can justify why Tia's equation is correct. Ask them to draw a representation to show the relationship, and, if necessary, show them the representation on the slide, which reveals the mathematical structure of the relationship.

Discuss the example with students and emphasise how this relates to the representation of the mathematical structure.

Explore/Discuss 1

Aim	To identify the mathematical structure of equations to understand their meaning
Materials	Mini whiteboards
Slides	Slide 13
Time	10 minutes

In this section of the lesson students explore equations involving variables that represent both the number and cost of wipes. Students are asked to write on their mini whiteboard what each of the equations means, providing an insight into students' understanding of **mathematical structure** (Key Principle 1) and **providing opportunities for their existing knowledge to be identified and built upon** (Key Principle 2). The use of equations with mathematical structures reflecting those used in the main activity helps to **build students' confidence and promote a culture where they believe that they can succeed** (Key Principle 5).

What the students might do and what you might do

Slide 13 Tell students that Tia's dad receives some multipacks of face wipes from the supplier. He receives two different types of face wipes; 'basic' wipes and 'deluxe' wipes.

Ask students to write on their mini whiteboards what equation (i) tells us about the face wipes received and to share their thinking. Check that students have identified the mathematical structure correctly and establish that twice as many 'deluxe' multipacks as 'basic' multipacks were received.

Show students equation (ii) and ask them what it tells us. In your discussion, emphasise that this equation gives us information about the *cost* of the wipes rather than the *number* ordered (as in equation (i)).

Show students the final equation and give them time to think about and write down what it means. Check that students understand that 'bz' means b multiplied by z and that the equation tells us that the cost of the 'basic' multipacks ordered is £12.

The focus of this discussion is the meaning of the equations, but you might also like to take this opportunity to check that your students can solve simple equations such as these but where there is only one unknown.

Explore 2

Aim	To explore the relationships between descriptions of scenarios and algebraic equations
Materials	'Completing descriptions' handout
Slides	Slides 14 and 15
Time	25 minutes

In this section of the lesson students are asked to write equations to represent a given scenario and interpret equations that have already been written. As the **connection between real-life situations and algebraic equations** is considered, students' understanding of how the use of algebra relates to mathematical relationships is developed (Key Principle 3).

What the students might do and what you might do

Slide 14 Tell students that Tia's dad has orders from his online shop that are ready to be sent out. Some of them are small parcels and some of them are large parcels. In the next activity they are going to be interpreting and writing equations that describe both the number of parcels and the cost of sending the parcels. Introduce the variables they will be using and emphasise the importance of checking that they are referring to number or cost as appropriate when completing the task.

Slide 15 Give each pair of students a copy of the 'Completing descriptions' handout. Explain that it costs £3 to send a small parcel. In their pairs, students need to fill in the missing statements or equations and use this information to work out how much it will cost Tia's dad to send the parcels. If you think your students will find this task overwhelming, ask them just to work on the first few rows.

As you circulate around the class, observe how the students are working and listen to their explanations to each other. Encourage students to check that they are using the appropriate variable for the *number* or *cost* of the parcels as they work. Get them to write down as much information as possible to show how they determine the total cost. Check also that students recognise that the cost of sending a large parcel will be greater than the cost of sending a small parcel (row D).

Discuss 2

Aim	To check students' understanding and discuss their thinking
Materials	Mini whiteboards
Slides	Slides 16–20
Time	15 minutes

Once students have had sufficient time to make progress, hold a class discussion. It is important to move beyond the answers and to explore students' thinking. Ask different pairs of students to explain what their thinking was when completing a particular row. It is important to **value different approaches** (Key Principle 2) and to **encourage all students to participate** in the class discussion (Key Principle 5).

What the students might do and what you might do

Slide 16 Discuss each of the rows in the table. There are different valid answers for each row. Take time to explore why different answers are correct. For example, some students may have used the fact that they know that $r = 3$ when completing the missing equations, as Tia has done. It is important that students recognise that this is also correct and can explain what Tia has done (substituted $r = 3$ in both rows B and F). Establish that it costs £9 to send a large parcel.

Slide 17 Remind students that to find the cost of sending the parcels we need to know the values of s and l (the number of small and large parcels being sent) as well as the cost for each size of parcel (r and t).

Ask students to consider the equation in Row F. Tell them that Tia uses this equation to find s.

Allow students some time to think about how to find s. You may want to ask them to write the value of s on their mini whiteboards. Then go through how Tia used the equation $3s = 12$ to determine that $s = 4$. Explore different ways of thinking about solving the equation to find s.

Slide 18 Ask students to consider the equation in Row A. Tell them that Tia uses this equation to find l. Allow students some time to think about how to find l. You may want to ask them to write the value of l on their mini whiteboards. Then go through how Tia used the equation $4 + l = 6$ to determine that $l = 2$. Explore different ways of thinking about solving the equation to find l.

Ask students which other row of the table could be used to find l when s is known (row E or row C) and establish that rows C and E provide the same information presented in different ways.

Slide 19 Once all four values have been identified, ask students to explain how they used the values to determine the cost of sending the small parcels (£12). Check that students recognise the need to multiply the number of small parcels to be sent (4) by the cost of sending a small parcel (£3).

Slide 20 Confirm that the cost of sending the large parcels is £18. Check whether students wrote an expression for the total cost. If they did not, ask them to write an expression for the total cost of sending the parcels on their mini whiteboards.

Review

Aim	To review students' understanding of mathematical structure
Materials	Mini whiteboards
Slides	Slides 21–23
Time	10 minutes

In this section of the lesson students **apply their knowledge** to a different context (Key Principle 4) to emphasise the importance of identifying and **understanding the mathematical structure** of a relationship (Key Principle 1).

What the students might do and what you might do

Slide 21 Bring students' thinking together by showing them a different scenario to emphasise the importance of understanding mathematical relationships.

Tell students that address labels come in sheets of 14 labels. Tia knows the cost of a sheet of address labels and wants to know how to work out the cost per label. Vinay tells her that she needs to multiply to get the cost per label, whereas Sade tells her to divide.

Ask students for a quick show of hands as to who they think is correct. Ask students to then write an equation on their mini whiteboards for the cost per label (f) given the cost of a sheet of labels (e). If students start to write an equation beginning with 'e =' rather than 'f =', remind them that what Tia wants to know is the cost per label (f).

After a couple of minutes ask students to show you their mini whiteboards. It is likely that students will either write the equation $f = 14e$ or $f = \frac{e}{14}$. If so, ask students to think about which equation makes sense if, for example, the cost of a sheet of address labels is £1.40. Will the cost per label be £1.40 multiplied by 14 or £1.40 divided by 14?

Establish that the cost per label is the cost of a sheet divided by 14, so Sade is correct and the equation needed is $f = \frac{e}{14}$.

Ask students to explain when you would need to multiply. Establish that if you knew the cost per label and wanted to work out the cost per sheet, then the cost per sheet is the cost per label multiplied by 14.

Slide 22 Emphasise that a mathematical relationship can be described in more than one way. If we want to know the cost per label (f) then we divide the cost of a sheet (e) by 14 ($f = \frac{e}{14}$).

We can also describe the relationship between the cost of a label and a sheet of labels in a different way, if we are interested in the cost of a sheet of address labels. Ask students for an equation to express the cost of a sheet of address labels (e) if the cost per label (f) is known and establish that $e = 14f$ also describes the relationship.

Whichever equation we are interested in, establish that in this relationship the cost per label is always the cost of a sheet divided by 14 and the cost of a sheet is the cost per label multiplied by 14.

Slide 23 Close the discussion by telling students that Sade, Vinay and Tia have all made incorrect suggestions for expressions relating to this scenario involving envelopes and address labels.

Ask students to look at the three suggested expressions and to explain why each one is incorrect. You may like to ask students to write the correct expressions on their mini whiteboards:

Total cost of envelopes ordered is hk (or kh)

Total number of envelope packs and address label sheets ordered is $k + g$

Total cost of all the envelopes and labels is $kh + ge$

Practice questions/Discuss 3

Aim	Students apply their knowledge to an unfamiliar task
Materials	‘Practice questions’ handout. It is not necessary to print this out: the question can be displayed on the board.
Slides	Slides 24–25
Time	10 minutes

Give each student a copy of the ‘Practice questions’ handout. Give students a couple of minutes to work on the questions individually and then discuss their thinking.

What the students might do and what you might do

Slides 24-25 Check whether students tried to introduce an additional variable to represent the total number of boats (question 1) or the total number of cups that Alison buys (question 2). Explain that this isn’t needed when writing an expression. Ask students whether they checked that their expressions worked by substituting some possible numbers in. If they did, ask them which numbers they chose and to explain how they used the result to determine whether or not their expressions were correct.

